ELEN 3401 - Exam 2 Equations Sheet
2025

Maxwell's Magnetostatics Equations

Gauss's Law for Magnetism

$$\nabla \cdot \mathbf{B} = 0 \quad \leftrightarrow \quad \oint_S \mathbf{B} \cdot d\mathbf{s} = 0$$

Ampère's Law

$$\nabla \times \mathbf{H} = \mathbf{J} \quad \leftrightarrow \quad \oint_C \mathbf{H} \cdot d\boldsymbol{\ell} = I$$

Lorentz Force on Charge q

$$\mathbf{F} = q(\mathbf{E} + \mathbf{u} \times \mathbf{B})$$

Magnetic Force on Wire

$$\mathbf{F}_{\mathrm{m}} = I \oint_C d\mathbf{l} \times \mathbf{B} \qquad \text{(N)}$$

Magnetic Torque on Loop

$$\mathbf{T} = \mathbf{m} \times \mathbf{B} \qquad \text{(N·m)}$$

$$\mathbf{m} = \hat{\mathbf{n}} NIA \qquad (\text{A·m}^2)$$

Biot–Savart Law

$$\mathbf{H} = \frac{I}{4\pi} \int_l \frac{d\mathbf{l} \times \hat{\mathbf{R}}}{R^2} \qquad \text{(A/m)}$$

Magnetic Field

Infinitely Long Wire $\quad \mathbf{B} = \hat{\boldsymbol{\phi}} \dfrac{\mu_0 I}{2\pi r} \quad (\text{Wb/m}^2)$

Circular Loop $\quad \mathbf{H} = \hat{\mathbf{z}} \dfrac{Ia^2}{2(a^2+z^2)^{3/2}} \quad \text{(A/m)}$

Solenoid $\quad \mathbf{B} \approx \hat{\mathbf{z}} \mu n I = \dfrac{\hat{\mathbf{z}} \mu N I}{l} \quad (\text{Wb/m}^2)$

Vector Magnetic Potential

$$\mathbf{B} = \nabla \times \mathbf{A} \qquad (\text{Wb/m}^2)$$

Vector Poisson's Equation

$$\nabla^2 \mathbf{A} = -\mu \mathbf{J}$$

Inductance

$$L = \frac{\Lambda}{I} = \frac{\Phi}{I} = \frac{1}{I} \int_S \mathbf{B} \cdot d\mathbf{s} \qquad \text{(H)}$$

Magnetic Energy Density

$$w_{\mathrm{m}} = \frac{1}{2} \mu H^2 \qquad (\text{J/m}^3)$$

Faraday's Law

$$V_{\mathrm{emf}} = -\frac{d\Phi}{dt} = -\frac{d}{dt} \int_S \mathbf{B} \cdot d\mathbf{s} = V_{\mathrm{emf}}^{\mathrm{tr}} + V_{\mathrm{emf}}^{\mathrm{m}}$$

Transformer

$$V_{\mathrm{emf}}^{\mathrm{tr}} = -N \int_S \frac{\partial \mathbf{B}}{\partial t} \cdot d\mathbf{s} \qquad (N \text{ loops}) \qquad \Phi = \int_S \vec{B} \cdot d\vec{s} \ [Wb]$$

Reference	Differential Form	Integral Form	
Gauss's law	$\nabla \cdot \mathbf{D} = \rho_{\mathrm{v}}$	$\oint_S \mathbf{D} \cdot d\mathbf{s} = Q$	(6.1)
Faraday's law	$\nabla \times \mathbf{E} = -\dfrac{\partial \mathbf{B}}{\partial t}$	$\oint_C \mathbf{E} \cdot d\mathbf{l} = -\int_S \dfrac{\partial \mathbf{B}}{\partial t} \cdot d\mathbf{s}$	(6.2)*
Gauss's law for magnetism	$\nabla \cdot \mathbf{B} = 0$	$\oint_S \mathbf{B} \cdot d\mathbf{s} = 0$	(6.3)
Ampère's law	$\nabla \times \mathbf{H} = \mathbf{J} + \dfrac{\partial \mathbf{D}}{\partial t}$	$\oint_C \mathbf{H} \cdot d\mathbf{l} = \int_S \left(\mathbf{J} + \dfrac{\partial \mathbf{D}}{\partial t}\right) \cdot d\mathbf{s}$	(6.4)

*For a stationary surface S.

$$\widetilde{\mathbf{E}}(z) = \hat{\mathbf{x}} \widetilde{E}_x^+(z) = \hat{\mathbf{x}} E_{x0}^+ e^{-jkz},$$

$$\widetilde{\mathbf{H}}(z) = \hat{\mathbf{y}} \frac{\widetilde{E}_x^+(z)}{\eta} = \hat{\mathbf{y}} \frac{E_{x0}^+}{\eta} e^{-jkz}.$$

$$\nabla^2 \tilde{E} - \gamma^2 \tilde{E} = 0$$

$$\gamma = \alpha + j\beta$$

Complex Permittivity

$$\epsilon_c = \epsilon' - j\epsilon''$$

$$\epsilon' = \epsilon$$

$$\epsilon'' = \frac{\sigma}{\omega}$$

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Maxwell's Equations for Time-Harmonic Fields

$$\nabla \cdot \widetilde{\mathbf{E}} = 0$$
$$\nabla \times \widetilde{\mathbf{E}} = -j\omega\mu\widetilde{\mathbf{H}}$$
$$\nabla \cdot \widetilde{\mathbf{H}} = 0$$
$$\nabla \times \widetilde{\mathbf{H}} = j\omega\epsilon_c\widetilde{\mathbf{E}}$$

Wave Polarization

$$\widetilde{\mathbf{H}} = \frac{1}{\eta}\,\hat{\mathbf{k}} \times \widetilde{\mathbf{E}}$$
$$\widetilde{\mathbf{E}} = -\eta\,\hat{\mathbf{k}} \times \widetilde{\mathbf{H}}$$

Power Density

$$\mathbf{S}_{av} = \tfrac{1}{2}\,\mathfrak{Re}\left[\widetilde{\mathbf{E}} \times \widetilde{\mathbf{H}}^*\right] \qquad (\mathrm{W/m^2})$$

	Any Medium	Lossless Medium ($\sigma = 0$)	Low-loss Medium ($\varepsilon''/\varepsilon' \ll 1$)	Good Conductor ($\varepsilon''/\varepsilon' \gg 1$)	Units
$\alpha =$	$\omega\left[\frac{\mu\varepsilon'}{2}\left[\sqrt{1+\left(\frac{\varepsilon''}{\varepsilon'}\right)^2}-1\right]\right]^{1/2}$	0	$\frac{\sigma}{2}\sqrt{\frac{\mu}{\varepsilon}}$	$\sqrt{\pi f\mu\sigma}$	(Np/m)
$\beta =$	$\omega\left[\frac{\mu\varepsilon'}{2}\left[\sqrt{1+\left(\frac{\varepsilon''}{\varepsilon'}\right)^2}+1\right]\right]^{1/2}$	$\omega\sqrt{\mu\varepsilon}$	$\omega\sqrt{\mu\varepsilon}$	$\sqrt{\pi f\mu\sigma}$	(rad/m)
$\eta_c =$	$\sqrt{\frac{\mu}{\varepsilon'}}\left(1-j\frac{\varepsilon''}{\varepsilon'}\right)^{-1/2}$	$\sqrt{\frac{\mu}{\varepsilon}}$	$\sqrt{\frac{\mu}{\varepsilon}}$	$(1+j)\frac{\alpha}{\sigma}$	(Ω)
$u_p =$	ω/β	$1/\sqrt{\mu\varepsilon}$	$1/\sqrt{\mu\varepsilon}$	$\sqrt{4\pi f/\mu\sigma}$	(m/s)
$\lambda =$	$2\pi/\beta = u_p/f$	u_p/f	u_p/f	u_p/f	(m)

Normal Incidence

$$\Gamma = \frac{E_0^r}{E_0^i} = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1}$$

$$\tau = \frac{E_0^t}{E_0^i} = \frac{2\eta_2}{\eta_2 + \eta_1}$$

$$\tau = 1 + \Gamma$$

$$\Gamma = \frac{\sqrt{\epsilon_{r_1}} - \sqrt{\epsilon_{r_2}}}{\sqrt{\epsilon_{r_1}} + \sqrt{\epsilon_{r_2}}} \quad (\text{if } \mu_1 = \mu_2)$$

$$S = \frac{|\tilde{E}_1|_{max}}{|\tilde{E}_1|_{min}} = \frac{1+|\Gamma|}{1-|\Gamma|}$$

$$\vec{S}_{av}^{\,i} = \hat{z}\,\frac{|E_0^i|^2}{2\eta_1}$$

$$\vec{S}_{av_1} = \vec{S}_{av}^{\,i} + \vec{S}_{av}^{\,r}$$

$$\vec{S}_{av}^{\,r} = -\hat{z}\,|\Gamma|^2\,\frac{|E_0^i|^2}{2\eta_1}$$

$$\vec{S}_{av}(z) = \frac{\hat{z}\left|\tilde{E}(0)\right|^2}{2|\eta_c|}\,e^{-2\alpha z}\cos\theta_\eta\ \left[\frac{W}{m^2}\right]$$

$$\frac{\tau^2}{\eta_2} = \frac{1-\Gamma^2}{\eta_1} \qquad \text{(lossless media)}$$

permittivity of free space	ε_0	$8.85 \times 10^{-12} \approx \frac{1}{36\pi} \times 10^{-9}$ F/m
permeability of free space	μ_0	$4\pi \times 10^{-7}$ H/m
intrinsic impedance of free space	η_0	$376.7 \approx 120\pi\ \Omega$